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STUDY OF THE HUMAN BODY- SEAT SYSTEM STABILITY

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Abstract. The paper presents the study of the human body, as a mechanical system, seated inside an auto vehicle and exposed under the vertical harmonic vibration action during the auto vehicle motion time. The human body may be roughly approximated by a linear lumped parameter at low frequencies and low vibration levels. Therefore, the model has 5 DOFs in translation, where 4 DOFs represents the human body and 1 DOF is for the seat cushion. The mechanical model of the human body, in sitting position on the seat cushion of a vehicle seat, consists of four parts: pelvis, upper torso, viscera and head. The eigenvalues for the human body – seat system was been calculate. The stability of a system is characterized by the eigenvalues of the system. The system stability analysis can also be given by the existence of a Lyapunov function for the system.

1. MECHANICAL MODEL OF THE HUMAN BODY / SEAT SYSTEM

This study developed a *linear biomechanical model* of the human body (HB) for evaluating the vibration transmissibility and dynamic response to vertical vibrations in sitting posture into an auto vehicle. Considering the human body as a *mechanical system*, at low frequencies (less than 100 Hz) and low vibration levels, it may be roughly approximated by linear lumped parameter systems. Therefore, the mechanical model has 5 DOFs in translation, where 4 DOFs represents the human body and 1 DOF is for the seat cushion (Fig. 1). The HB lumped parameter systems comprising masses $\mathbf{M_i}$, springs $\mathbf{K_i}$ and dampers $\mathbf{C_i}$ for i=2, 3, 4 and 5, corresponding to the following segments of the human body: *pelvis, upper torso, viscera* and *head*, respectively. The soft seat cushion was implemented as a linear spring K_{2C} and damper C_{2C} system. The seat is fixed to the floor through the seat suspension which is formatted by spring and dashpot and is represented by spring K_1 and damping C_1.

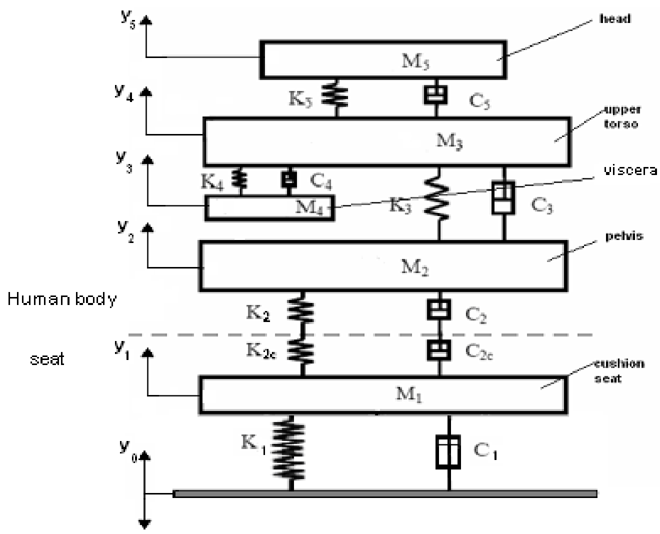

Fig. 1 The human body / seat system

2. MATHEMATICAL MODEL. THE MOTION EQUATIONS OF THE SYSTEM

The equations of motion (from the static equilibrium position) for the HB/S mechanical model can be derived as follows:

$$M_1\ddot{y}_1 + K_1(y_1 - y_s) + K_{2t}(y_1 - y_2) + C_1(\dot{y}_1 - \dot{y}_s) + C_{2t}(\dot{y}_1 - \dot{y}_2) = 0$$
$$M_2\ddot{y}_2 + K_{2t}(y_2 - y_1) + C_{2t}(\dot{y}_2 - \dot{y}_1) - K_3(y_3 - y_2) - C_3(\dot{y}_3 - \dot{y}_2) = 0$$
$$M_3\ddot{y}_3 + K_3(y_3 - y_2) + C_3(\dot{y}_3 - \dot{y}_2) + K_4(y_3 - y_4) + C_4(\dot{y}_3 - \dot{y}_4) - K_5(y_5 - y_3) - C_5(\dot{y}_5 - \dot{y}_3) = 0$$
$$M_4\ddot{y}_4 + K_4(y_4 - y_3) + C_4(\dot{y}_4 - \dot{y}_3) = 0$$
$$M_5\ddot{y}_5 + K_5(y_5 - y_3) + C_5(\dot{y}_5 - \dot{y}_3) = 0$$
(1)

where y_s represents the displacement excitation, with $y_s = y_0 \sin \omega t$, $\dot{y}_s = \omega y_0 \cos \omega t$, y_1 represents the displacement response of seat cushion M_1, y_i, i=2÷5 represent the displacement response of the body segments M_i, and

$$K_{2t} = \frac{K_2 K_{2c}}{K_2 + K_{2c}}, \quad C_{2t} = \frac{C_2 C_{2c}}{C_2 + C_{2c}} \qquad (2)$$

3. STABILITY OF THE MODEL

There are many theories that apply to the stability of MDOF systems. The most common method of analyzing the stability of such systems is to show the existence of a Lyapunov function for the system.

3.1. Lyapunov stability

One of the most widely adopted stability concepts is Lyapunov stability, which plays important roles in system and control theory and in the analysis of engineering systems. This is the primary method of testing the stability of linear systems with uncertainly or reliability problems. To determine stability or instability of the system Lyapunov introduced two main methods: **Lyapunov's *first* or *indirect* method, Lyapunov's *second* or *direct* method.** Lyapunov's direct method allows us to determine the stability of a linear system without explicitly integrating the differential equation. The method is a generalization of the idea that if there is some "*measure of energy*" in a system, then we can study the rate of change of the energy of the system to ascertain stability. It involves finding a Lyapunov function for a system. If such a function exists, than the system is stable.

A Lyapunov function, denoted by V(x), is a real scalar function of the vector x(t), which has continuous first partial derivatives and satisfies the following conditions:

1) $V(x)$ **> 0** for all values of x(t) ≠ 0;

2) $\dot{V}(x)$ **< 0** for all values of x(t) ≠ 0.

where $\dot{V}(x)$ denotes the time derivative of the function V(x). For the **5-DOF** model, $x(t) = [x_1(t), x_2(t), x_3(t), x_4(t), x_5(t)]$.

Based on the definition of a Lyapunov function, several extremely useful stability theorems can be stated. The first result states that if there exists a Lyapunov function for a given system, then that system is stable. If, in addition, the function *V* (x) is strictly less than zero, the system is asymptotically stable. It should be noted that if a Lyapunov function cannot be found, nothing can be concluded about the stability of the system, as the Lyapunov theorems are only sufficient conditions for stability.

3.2. Numerical model. Lyapunov's Direct Method

We would like to investigate the stability of this system using Lyapunov theory. An equivalent state space representation of the system's equations of motion (1) is given by the state model (2) for where **A** is (10×10) system matrix.

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t) \quad (2)$$

The eigenvalues of the system matrix **A** was been calculate.

- -2.5725e+001 +2.1378e+002i
- -2.5725e+001 -2.1378e+002i
- -2.2018e+001 +5.8960e+001i
- -2.2018e+001 -5.8960e+001i
- -1.5507e+001 +5.2261e+001i
- -1.5507e+001 -5.2261e+001i
- -1.8638e+000 +1.2961e+001i
- -1.8638e+000 -1.2961e+001i
- -1.1586e+001 +1.9007e+001i
- -1.1586e+001 -1.9007e+001i

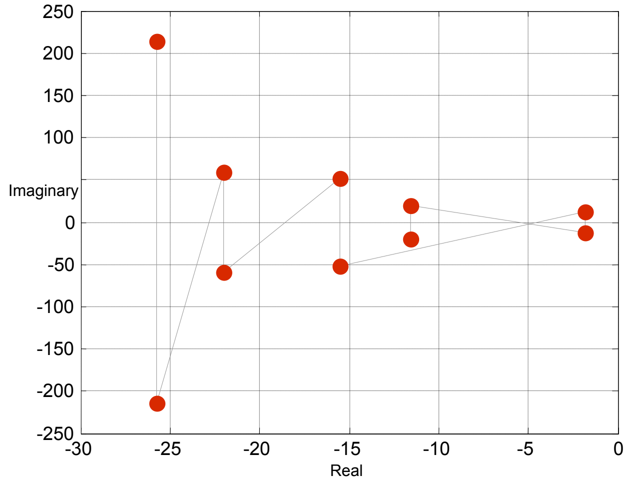

Fig. 2 The eigenvalues of matrix A

All the eigenvalues of matrix **A** have negative real parts.

For the Lyapunov Equation $\boldsymbol{A^TP + PA + Q = 0}$ we are given **A** and **Q** (positive definite matrix) and want to find ***P*** (positive definite matrix). The standard "initial guess" for Q is identity, i.e. Q = I_{10}. Using the MATLAB function **lyap** we can execute the following statement P = lyap (A^T, Q) and obtain the solution for P, where P is a symmetric (10×10) matrix. If ***P*** is positive definiteness matrix with all eigenvalues in the closed right half plane, the Lyapunov test indicates that the system under consideration is ***asymptotically stable.***

- 1.2310e+292
- 1.5495e+295
- 3.8949e+296
- 1.0333e+298
- 3.0842e+298
- 1.4782e+299
- 5.8844e+299
- 9.9453e+300
- 5.0438e+301
- 1.6360e+302

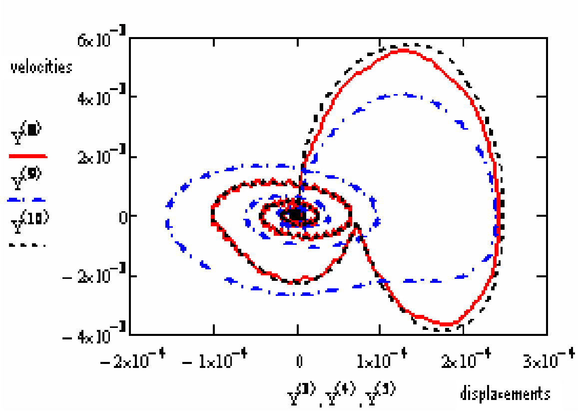

Fig. 3 Stability system in the sense of Lyapunov

Finding Lyapunov functions **V(x)** in general is very difficult. For linear system with specific choice of look ahead distance: potential energy plus kinetic energy is Lyapunov function. Lyapunov function is an energy-like function of the system positional states meeting two requirements:

- $V(x)$ function is positive for all system states;
- $\dot{V}(x)$ function decreases along system trajectories.

Using the quadratic form for E (total energy) of the HB/S system and $\dot{E}$ (change in energy over time), by analogy, we can identify **the Lyapunov function** like $V = E$, where $V = x^T \cdot P \cdot x$, and $\dot{V} = \dot{E}$, where $\dot{V} = -x^T Q x$. The found Lyapunov function satisfies the both conditions: V>0 and $\dot{V}(x)$<0 (change in energy was caused by motion of system). In conclusion V (x) shows that system energy can't increase; it must be stable.

The stability of a system can also be characterized by the eigenvalues of the system, which is similar to the existence of a Lyapunov function. In fact, it can be easily shown that a given linear system (1) is stable if and only if it has no eigenvalues with positive real part. Moreover, the system will be asymptotically stable if and only if all of its eigenvalues have negative real parts (no zero parts allowed). The eigenvalues approach to stability has the attraction of being both necessary and sufficient.

For the HB / S system (1) the eigenvalues of the matrix are the following:

$$\lambda_{1,2} = -4.094 \pm 34.033i$$
$$\lambda_{3,4} = -3.504 \pm 9.379i$$
$$\lambda_{5,6} = -2.469 \pm 8.321i$$
$$\lambda_{7,8} = -1.844 \pm 3.052i$$
$$\lambda_{9,10} = -0.296 \pm 2.063i$$

Consequently, the 5-DOF model of human body / seat system is asymptotically stable since the real parts of all of tenth eigenvalues are negative.

4. CONCLUSION

The stability analysis is given by the existence of a Lyapunov function for the HB/S system. If, in addition, the function V (x) is strictly less than zero, the system is asymptotically stable. In this case the HB / seat system is stable in sense of Lyapunov.

The eigenvalues of the system can also give the stability of the system.

Comparing with other methods, such as Root Locus Analysis, Nyquist Stability Graphs, Bode Graphs, Frequency (Harmonic) Response Methods and others, the Lyapunov method has the following advantages: the solving of motion's differential equations is not necessary; it is more direct; it is simpler for LTI system; it is more understandable.

The mechanical model of the human body / seat system, proposed in this paper is a stable linear system, which will be used to study the biodynamic responses of HB using some different seat suspensions.

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